9A

1(a) $\frac{d}{dx}(x^8)$

$= 8x^7$

(c) $\frac{d}{dx}(x)$

$= 1$

(e) $\frac{d}{dx}(x^{-6})$

$= -6x^{-7}$

(g) $\frac{d}{dx}(4)$

$= 0$

(i) $\frac{d}{dx}\left(\frac{3}{4}x^8\right)$

$= 8 \times \frac{3}{4}x^7$

$= 6x^7$

(k) $\frac{d}{dx}(-5)$

$= 0$

(m) $\frac{d}{dx}(x^{3/2})$

$= \frac{3}{2}x^{1/2}$

(o) $\frac{d}{dx}(x^{7/5})$

$= \frac{7}{5}x^{2/5}$

(q) $\frac{d}{dx}x^{1/4}$

$= \frac{1}{4}x^{-3/4}$

(r) $\frac{d}{dx}(x^{3/3})$

$= \frac{2}{3}x^{-1/3}$

(t) $\frac{d}{dx}(-x^{-2/3})$

$= \frac{2}{3}x^{-5/3}$

9A

1(v) $\frac{d}{dx}\left(6x^{-\frac{1}{3}}\right)$

$= -\frac{1}{3} \times 6x^{-\frac{4}{3}}$

$= -2x^{-\frac{4}{3}}$

1(x) $\frac{d}{dx}\left(-\frac{1}{4}\right)$

$= 0$

2(a) $\frac{d}{dx}\left(x^2 + 2x - 1\right)$

$= 2x + 2$

(c) $\frac{d}{dx}\left(3x - 4\right)$

$= 3$

(e) $\frac{d}{dx}\left(\frac{1}{3}x^3 + 2x^2 - 12x + 4\right)$

$= 3 \times \frac{1}{3}x^2 + 4x - 12$

$= x^2 + 4x - 12$

(g) $\frac{d}{dx}\left(-5x^3 - 2x^{-6}\right)$

$= 3 \times -5x^2 - (-6) \times 2x^{-7}$

$= -15x^2 + 12x^{-7}$

(i) $\frac{d}{dx}\left(\frac{1}{2}x^4 + 2x - 3x^{-2}\right)$

$= 4 \times \frac{1}{2}x^3 + 2 - (-2) \times 3x^{-3}$

$= 2x^3 + 2 + 6x^{-3}$

9A

3(b) $\frac{d}{dx}\left(12x^{3/2} - x^{5/2}\right)$

$= \frac{3}{2} \times 12x^{1/2} - \frac{5}{2}x^{3/2}$

$= 18x^{1/2} - \frac{5}{2}x^{3/2}$

(d) $\frac{d}{dx}\left(\frac{2}{5}x^{5/4} - 3x^{-1/3}\right)$

$= \frac{5}{4} \times \frac{2}{5}x^{1/4} - \left(-\frac{1}{3}\right) \times 3x^{-4/3}$

$= \frac{1}{2}x^{1/4} + x^{-4/3}$

(f) $\frac{d}{dx}\left(\frac{2}{3}x^{-6} + 8x^{1/4}\right)$

$= -6 \times \frac{2}{3}x^{-5} + \frac{1}{4} \times 8x^{-3/4}$

$= -4x^{-5} + 2x^{-3/4}$

(h) $\frac{d}{dx}\left(\frac{4}{5}x^{1/4} - \frac{5}{4}x^{-1/5}\right)$

$= \frac{1}{4} \cdot \frac{4}{5}x^{-3/4} - \left(-\frac{1}{5}\right)\frac{5}{4}x^{-6/5}$

$= \frac{1}{5}x^{-3/4} + \frac{1}{4}x^{-6/5}$

9A

4a) $y = x^3 - 4x^2 + x - 8$

$\frac{dy}{dx} = 3x^2 - 8x + 1$

b) $f(x) = x^{4/3}$

$f'(x) = \frac{4}{3} x^{1/3}$

c) $y = 6x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2} 6x^{-1/2}$

$= 3x^{-1/2}$

d) $f(x) = 2x^2 - 3x - 4$

$f'(x) = 4x - 3$

e) $\frac{d}{dx}(x^{3/4} - 8x)$

$= \frac{3}{4} x^{-1/4} - 8$

9A

5a) $\frac{d}{dp}(p^4 - 6p^2 + 9)$

$= 4p^3 - 12p^2$

(c) $\frac{d}{dw}(w^3 - 3w + 6)$

$= 3w^2 - 3$

(e) $\frac{d}{dt}(8t^{3/2} - 4t^{-1/2})$

$= \frac{3}{2} \times 8t^{1/2} - (-\frac{1}{2})4t^{-3/2}$

$= 12t^{1/2} + 2t^{-3/2}$

(g) $\frac{d}{dt}(6t^{-4} - 8t + 7)$

$= -4 \times 6t^{-5} - 8$

$= -24t^{-5} - 8$

9B

3(a) $y = (x-3)(x+5)$

$y = x^2 + 5x - 3x - 15$

$y = x^2 + 2x - 15$

$\frac{dy}{dx} = 2x + 2$

(c) $y = x(x+3)(x-2)$

$y = x(x^2 - 2x + 3x - 6)$

$y = x^3 + x^2 - 6x$

$\frac{dy}{dx} = 3x^2 + 2x - 6$

(e) $g(x) = 2x^2(x-1)^2$

$= 2x^2(x-1)(x-1)$

$= 2x^2(x^2 - 2x + 1)$

$= 2x^4 - 4x^3 + 2x$

$= 8x^3 - 12x^2 + 2$

9B

3(g) $y = (x+4)(x+1)(x-2)$

$y = (x^2 + 5x + 4)(x-2)$

$y = x^3 + 5x^2 + 4x - 2x^2 - 10x - 8$

$y = x^3 + 3x^2 - 6x - 8$

$\frac{dy}{dx} = 3x^2 + 6x - 6$

4(a) $y = \frac{5}{x^2}$

$y = 5x^{-2}$

~~$\frac{d}{dx}(5x^{-2})$~~

$\frac{dy}{dx} = -10x^{-3}$

$\frac{dy}{dx} = \frac{-10}{x^3}$

(c) $y = \frac{1}{2x^3}$

$y = \frac{x^{-3}}{2}$

~~$\frac{d}{dx}\left(\frac{x^{-3}}{2}\right)$~~

$\frac{dy}{dx} = \frac{-3x^{-4}}{2} = \frac{-3}{2x^4}$

9B

4(e) $g(x) = 4x^3 - \frac{2}{x^5}$

$g(x) = 4x^3 - 2x^{-5}$

$g'(x) = 12x^2 - (-5) \times 2x^{-6}$

$= 12x^2 + 10x^{-6}$

$= 12x^2 + \frac{10}{x^6}$

4(i) $f(x) = \frac{3}{2x^4} - 5x - 6$

$f(x) = \frac{3x^{-4}}{2} - 5x - 6$

$f'(x) = \frac{-12x^{-5}}{2} - 5$

$= \frac{-6}{x^5} - 5$

9B

(5) (a) $y = 8\sqrt{x}$

$= 8x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} * 8x^{-\frac{1}{2}}$

$= \frac{4}{x^{\frac{1}{2}}}$

$= \frac{4}{\sqrt{x}}$

(c) $f(x) = 12\sqrt[4]{x^3}$

$= 12x^{4/3}$

$f'(x) = \frac{4}{3} \times 12x^{\frac{1}{3}}$

$= 16\sqrt[3]{x}$

(e) $y = \sqrt{x^7}$

$= x^{\frac{7}{2}}$

$\frac{dy}{dx} = \frac{7}{2}x^{5/2}$

$= \frac{7\sqrt{x^5}}{2}$

9B

5(g) $y = \frac{1}{x\sqrt{x}}$

$= \frac{1}{x^1 x^{1/2}}$

$= \frac{1}{x^{3/2}}$

$= x^{-3/2}$

$\frac{dy}{dx} = \frac{-3x^{-5/2}}{2}$

$= \frac{-3}{2x^{5/2}}$

$= \frac{-3}{2\sqrt{x^5}}$

5(i) $y = \frac{10}{\sqrt{x^3}}$

$= \frac{10}{x^{3/2}}$

$= 10x^{-3/2}$

$\frac{dy}{dx} = \frac{-3}{2} \times 10x^{-5/2}$

$= -15x^{-5/2}$

$= \frac{-15}{x^{5/2}}$

$= \frac{-15}{\sqrt{x^5}}$

9B

5(ii) $y = \frac{3}{2}\left(\sqrt[9]{x^4}\right)$

$y = \frac{3}{2} x^{4/9}$

$\frac{4}{9} - 1$

$= \frac{4}{9} - \frac{9}{9}$

$= -\frac{5}{9}$

$\frac{dy}{dx} = \frac{3}{2} \times \frac{4}{9} x^{-5/9}$

$= \frac{2}{3} x^{-5/9}$

$= \frac{2}{3x^{5/9}}$

$= \frac{2}{3\sqrt[9]{x^5}}$

9B

5(m) $g(x) = \dfrac{5}{2\sqrt[3]{x^2}}$

$= \dfrac{5}{2x^{2/3}}$

$= \dfrac{5x^{-2/3}}{2}$

$g'(x) = -\dfrac{2'}{3} \cdot \dfrac{5}{x'} x^{-5/3}$

$= -\dfrac{5}{3} x^{-5/3}$

$= \dfrac{-5}{3x^{5/3}}$

$= \dfrac{-5}{3\sqrt[3]{x^5}}$

9B

6(a) $y = \frac{x^2 - 4}{x}$

$y = \frac{x^2}{x} - \frac{4}{x}$

$y = x - 4x^{-1}$

$\frac{dy}{dx} = 1 - (-1)4x^{-2}$

$\frac{dy}{dx} = 1 + \frac{4}{x^2}$

(c) $f(x) = \frac{4 - x^3}{x}$

$= \frac{4}{x} - \frac{x^3}{x}$

$= 4x^{-1} - x^2$

$f'(x) = -4x^{-2} - 2x$

~~$= 8bx^{-3}$~~

$= -\frac{4}{x^2} - 2x$

9B

6(e) $y = \dfrac{1 - x^4}{2x^3}$

$= \dfrac{1}{2x^3} - \dfrac{x^4}{2x^3}$

$= \dfrac{x^{-3}}{2} - \dfrac{x}{2}$

$\dfrac{dy}{dx} = \dfrac{-3x^{-4}}{2} - \dfrac{1}{2}$

$= \dfrac{-3}{2x^4} - \dfrac{1}{2}$

6(g) $y = \dfrac{(x+1)(x+4)}{x^2}$

$= \dfrac{x^2 + 5x + 4}{x^2}$

$= \dfrac{x^2}{x^2} + \dfrac{5x}{x^2} + \dfrac{4}{x^2}$

$= 1 + \dfrac{5}{x} + \dfrac{4}{x^2}$

$= 1 + 5x^{-1} + 4x^{-2}$

$\dfrac{dy}{dx} = -5x^{-2} - 8x^{-3}$

$= \dfrac{-5}{x^2} - \dfrac{8}{x^3}$

9B

6(i) $y = \dfrac{(x-2)(2x+1)^2}{x^3}$

$= \dfrac{(x-2)(2x+1)(2x+1)}{x^3}$

$= \dfrac{(2x^2-3x-2)(2x+1)}{x^3}$

$= \dfrac{4x^3-6x^2-4x+2x^2-3x-2}{x^3}$

$= \dfrac{4x^3-4x^2-7x-2}{x^3}$

$= \dfrac{4x^3}{x^3} - \dfrac{4x^2}{x^3} - \dfrac{7x}{x^3} - \dfrac{2}{x^3}$

$= 4 - \dfrac{4}{x} - \dfrac{7}{x^2} - \dfrac{2}{x^3}$

$= 4 - 4x^{-1} - 7x^{-2} - 2x^{-3}$

$\dfrac{dy}{dx}$ ↑

$\dfrac{dy}{dx} = 4x^{-2} + 14x^{-3} + 6x^{-4}$

$= \dfrac{4}{x^2} + \dfrac{14}{x^3} + \dfrac{6}{x^4}$

9B

7(a) $y = \sqrt{x}(x-3)$

$= x^{\frac{1}{2}}(x-3)$

$= x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2} \cdot 3x^{-\frac{1}{2}}$

$= \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2} - \frac{3}{2\sqrt{x}}$

(8)(c) $g(x) = \left(1-\sqrt{x}\right)\left(2-\frac{1}{\sqrt{x}}\right)$

$= 2 - \frac{1}{\sqrt{x}} - 2\sqrt{x} + \frac{\sqrt{x}}{\sqrt{x}}$

$= 2 - x^{-\frac{1}{2}} - 2x^{\frac{1}{2}} + 1$

$= 3 - x^{-\frac{1}{2}} - 2x^{\frac{1}{2}}$

$g'(x) = \frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{2} \times 2x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}}$

Q8

7(e) $y = \left(\frac{1}{2x} - 2\sqrt{x}\right)\left(\frac{2}{\sqrt{x}} - x\right)$

$$y = \frac{1}{4x\sqrt{x}} - \frac{x}{2x} - \frac{4\sqrt{x}}{\sqrt{x}} + 2x\sqrt{x}$$

$$= x^{-3/2} - \frac{1}{2} - 4 + 2x^{3/2} \qquad \left(x\sqrt{x} = x \cdot x^{1/2} = x^{3/2}\right)$$

$$= x^{-3/2} - \frac{9}{2} + 2x^{3/2}$$

$$\frac{dy}{dx} = -\frac{3}{2}x^{-5/2} + \frac{3}{2} \cdot 2x^{1/2}$$

$$= \frac{-3}{2\sqrt{x^5}} + 3\sqrt{x}$$

8(a) $f(x) = \dfrac{2 - x^3}{\sqrt{x}}$

$$= \frac{2}{\sqrt{x}} - \frac{x^3}{\sqrt{x}}$$

$$= 2x^{-1/2} - x^{5/2}$$

$$f'(x) = -\frac{1}{2} \cdot 2x^{-3/2} - \frac{5}{2}x^{3/2}$$

$$= -x^{-3/2} - 5x^{3/2}$$

$$= \frac{-1}{\sqrt{x^3}} - 5\sqrt{x^3}$$

9B

8(c) $g(x) = \dfrac{\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x}\right)}{2x^2}$

$$g(x) = \frac{1 - \frac{2}{x} - \frac{1}{x} + \frac{2}{x^2}}{2x^2}$$

$$= \frac{1 - \frac{3}{x} + \frac{2}{x^2}}{2x^2}$$

$$= \frac{1}{2x^2} - \frac{3}{2x^3} + \frac{2}{2x^4}$$

$$= \frac{x^{-2}}{2} - \frac{3x^{-3}}{2} + \frac{2x^{-4}}{2}$$

$$g'(x) = -\frac{2x^{-3}}{2} + \frac{9x^{-4}}{2} - \frac{8x^{-5}}{2}$$

$$= \frac{-2}{2x^3} + \frac{9}{2x^4} - \frac{8}{2x^5}$$

$$= -\frac{1}{x^3} + \frac{9}{2x^4} - \frac{4}{x^5}$$

9B

8(e) $f(x) = \dfrac{\left(\frac{4}{x} - \frac{3}{2x}\right)^2}{4\sqrt[3]{x^2}}$ $\qquad \frac{4}{x} = \frac{8}{2x}$

$$= \frac{\left(\frac{8}{2x} - \frac{3}{2x}\right)^2}{4\sqrt[3]{x^2}}$$

$$= \frac{\left(\frac{5}{2x}\right)^2}{4\sqrt[3]{x^2}}$$

$$= \frac{\frac{25}{4x^2}}{4x^{2/3}}$$

$$= \frac{25}{16x^2 \cdot x^{2/3}}$$

$$= \frac{25}{16x^{8/3}}$$

$$= \frac{25x^{-8/3}}{16}$$

$$f'(x) = -\frac{8}{3} \cdot \frac{25}{16} x^{-11/3} = -\frac{25}{6} x^{-11/3} = \frac{-25}{6\sqrt[3]{x^{11}}}$$

9D

1(a) $y = x^2 - 2x + 4$

$$\frac{dy}{dx} = 2x - 2$$

$$\frac{dy}{dx}(3) = 2(3) - 2$$

$$= 4$$

$m = 4$ at $(3, 7)$

(c) $y = -2 + 2x - x^2$

$$\frac{dy}{dx} = 2 - 2x$$

$$\frac{dy}{dx}(0) = 2 - 2(0)$$

$$= 2$$

$m = 2$ at $(0, -2)$

2(a) $y = x^2 + 4x + 2$

$$\frac{dy}{dx} = 2x + 4$$

$$\frac{dy}{dx}(3) = 2(3) + 4$$

$$= 10$$

$m = 10$

9D

2(c) $f(x) = x^3 - 4x^2 + 5x + 3$

$f'(x) = 3x^2 - 8x + 5$

$f'(1) = 3(1)^2 - 8(1) + 5$

$= 0$

2(e) $g(x) = 6x - x^3$

$g'(x) = 6 - 3x^2$

~~$g'(x)$~~

$g'(-2) = 6 - 3(-2)^2$

$= 6 - 12$

$= -6$

3 ~~(a)~~ $y = \frac{2}{x}$

~~$\frac{dy}{}$~~

$y = 2x^{-1}$

$\frac{dy}{dx} = -2x^{-2}$

$= \frac{-2}{x^2}$

(a) $\frac{dy}{dx}(1) = \frac{-2}{(1)^2} = -2$ (b) $\frac{dy}{dx}(-3) = \frac{-2}{(-3)^2} = -\frac{2}{9}$

(c) $\frac{dy}{dx}\left(\frac{1}{2}\right) = \frac{-2}{\left(\frac{1}{2}\right)^2} = \frac{-2}{\frac{1}{4}} = -8$

9D

(6) $y = \frac{5}{4x^2}$

$= \frac{5x^{-2}}{4}$

$\frac{dy}{dx} = -2 \cdot \frac{5x^{-3}}{4}$

$= \frac{-10}{4x^3}$

$= \frac{-5}{2x^3}$

$\frac{dy}{dx} = \frac{-5}{2(-10)^3}$

$= \frac{-5}{-2000}$

$= \frac{1}{400}$

9) 7(a)

$$y = x^2 + 8x - 3$$

$$\frac{dy}{dx} = 2x + 8$$

$$m = \frac{dy}{dx} = 2$$

$$2x + 8 = 2$$

$$2x = -6$$

$$x = -3$$

7(b) $f(x) = 5 - 4x - x^2$

$$f'(x) = -4 - 2x$$

$f'(p) = 2 \Rightarrow -4 - 2p = 2$

$$-2p = 6$$

$$p = \frac{6}{-2}$$

$$p = -3$$

9D

10 (a) $y = \frac{1}{3}x^3 - 3x^2 + 12x + 2$

$$\frac{dy}{dx} = x^2 - 6x + 12$$

$m = \frac{dy}{dx} = 4 \Rightarrow x^2 - 6x + 12 = 4$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$x - 4 = 0 \quad x - 2 = 0$

$x = 4, \; x = 2$

10 (b) $y = x^3 + 2x^2 - 7x + 1$

$$\frac{dy}{dx} = 3x^2 + 4x - 7$$

$m = \frac{dy}{dx} = -3 \Rightarrow 3x^2 + 4x - 7 = -3$

$$3x^2 + 4x - 4 = 0$$

$$(3x - 2)(x + 2) = 0$$

$3x - 2 = 0 \qquad x + 2 = 0$

$3x = 2 \qquad x = -2$

$x = \frac{2}{3}$

9D

(14) $x - \frac{1}{4}y + 2 = 0$ (Put in form $y = mx + c$)

or

$-\frac{1}{4}y = -x - 2$

$y = -4(-x - 2)$

$y = 4x + 8$ $\quad M = 4$

$x + 2 = \frac{1}{4}y$

$4(x + 2) = y$

$y = 4x + 8$

$y = x^2 - 2x + 6$

$\frac{dy}{dx} = 2x - 2$

as parallel to line $m = \frac{dy}{dx} = 4$

$2x - 2 = 4$

$2x = 6$

$x = 3$

$y(3) = 3^2 - 2(3) + 6$

$= 9$

$(3, 9)$

(Note, answer in textbook is different)

9D

(12) $y = \frac{8}{x\sqrt{x}}$

$= \frac{8}{x \cdot x^{1/2}}$

$= \frac{8}{x^{3/2}}$

$= 8x^{-3/2}$

$\frac{dy}{dx} = -\frac{3}{2} \cdot 8x^{-5/2}$

$= -12x^{-5/2}$

$= \frac{-12}{\sqrt{x^5}}$

$M = \frac{dy}{dx} = -\frac{3}{8} \Rightarrow \frac{-12}{\sqrt{x^5}} = \frac{-3}{8}$

$-12 \times 8 = -3\sqrt{x^5}$

$\frac{-96}{-3} = \sqrt{x^5}$

$24 = \sqrt{x^5}$

$24^2 = x^5$

$\sqrt[5]{24^2} = x$

$x =$

9)

(16)(a) $f(x) = (x-4)^2 + 1$

$= (x-4)(x-4) + 1$

$= x^2 - 8x + 16 + 1$

$= x^2 - 8x + 17$

$f'(x) = 2x - 8$

at Q $m = 4 \Rightarrow 2x - 8 = 4$

$2x = 12$

$x = 6$

$y(6) = (6-4)^2 + 1$

$= 2^2 + 1$

$= 5$

Q $(6, 5)$

(b) $m = -4$

$2x - 8 = -4$

$2x = 4$

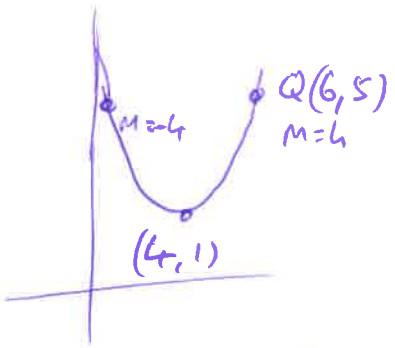
$x = 2$

$y(2) = (2-4)^2 + 1$

$= 5$

$(2, 5)$

or



by symmetry when $m = -4$ point is $(+2, 5)$

9D

16(c) $h(x) = f(x-2) - 5$

$x \to x+2$

$y \to y-5$

$(2, 5) \to (4, 0)$

9E

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

1 (b) $\frac{d}{dx}(3\cos x)$

$= -3\sin x$

(e) $\frac{d}{dx}\left(\frac{2}{3}\cos x\right)$

$= -\frac{2}{3}\sin x$

(h) $\frac{d}{dx}(3\sin x + 7\cos x)$

$= 3\cos x - 7\sin x$

(k) $\frac{d}{dx}\left(\frac{3}{x^2} - \cos x\right)$

$= \frac{d}{dx}(3x^{-2} - \cos x)$

$= -6x^{-3} - -\sin x$

$= -\frac{6}{x^3} + \sin x$

9E

1(m) $\frac{d}{dx}\left(5x^3 - \frac{1}{\sqrt[3]{x^5}} + 9\sin x\right)$

$= \frac{d}{dx}\left(5x^3 - x^{-5/3} + 9\sin x\right)$

$= 15x^2 + \frac{5}{3}x^{-8/3} + 9\cos x$

$= 15x^2 + \frac{5}{3\sqrt[3]{x^8}} + 9\cos x$

1(q) $\frac{d}{dx}\left(\frac{6}{\sqrt{x}} - 4\sin x\right)$

$= \frac{d}{dx}\left(6x^{-1/2} - 4\sin x\right)$

$= -\frac{1}{2} \cdot 6x^{-3/2} - 4\cos x$

$= -3x^{-3/2} - 4\cos x$

$= \frac{-3}{\sqrt{x^3}} - 4\cos x$

9E

2(a) $f(x) = 6\sin x$

$f'(x) = 6\cos x$

$f'\left(\frac{\pi}{3}\right) = 6\cos\frac{\pi}{3}$

$= 6 \times \frac{1}{2}$

$= 3$

(c) $y = \frac{1}{2}\sin x$

$\frac{dy}{dx} = \frac{1}{2}\cos x$

$\frac{dy}{dx}\left(\frac{\pi}{4}\right) = \frac{1}{2}\cos\frac{\pi}{4}$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}$

9E

Q1)

3(a)

$y = 4\sin x$

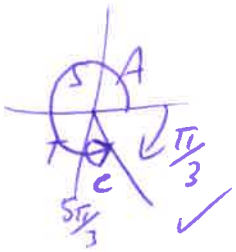
$\frac{dy}{dx} = 4\cos x$

$\frac{dy}{dx}\left(\frac{5\pi}{3}\right) = 4\cos\frac{5\pi}{3}$

$= 4\cos\frac{\pi}{3}$

$= 4 \times \frac{1}{2}$

$= 2$



$\frac{5\pi}{3} = 300°$

$\frac{\pi}{3} = 60°$

$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

3(c) $y = \frac{3}{4}\sin x$

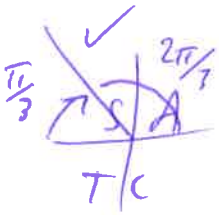
$\frac{dy}{dx} = \frac{3}{4}\cos x$

$\frac{dy}{dx}\left(\frac{2\pi}{3}\right) = \frac{3}{4}\cos\left(\frac{2\pi}{3}\right)$

$= -\frac{3}{4}\cos\left(\frac{\pi}{3}\right)$

$= -\frac{3}{4} \cdot \frac{1}{2}$

$= -\frac{3}{8}$



$\frac{2\pi}{3} = 120$

$\frac{\pi}{3} = 60$

$\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

9E

5(a) $y = 3\sin x - 4$

$\frac{dy}{dx} = 3\cos x$

$\frac{dy}{dx}(2) = 3\cos(2)$

$= \underline{-1.25}$ *

(c) $y = 7\sin x - 5\cos x$

$\frac{dy}{dx} = 7\cos x + 5\sin x$

$\frac{dy}{dx}(2.2) = 7\cos(2.2) + 5\sin(2.2)$

$= \underline{-0.08}$

9E

(7)(a) $y = 8\cos x$

$$\frac{dy}{dx} = -8\sin x$$

$$\frac{dy}{dx} = -4 \Rightarrow -8\sin x = -4$$

$$\sin x = \frac{-4}{-8}$$

$$\sin x = \frac{1}{2}$$

$\pi -$ ✓ | ✓

S | A

T | C

$$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

(9)(b) $y = \frac{1}{2}\sin x$

$$\frac{dy}{dx} = \frac{1}{2}\cos x$$

$m = \frac{\sqrt{2}}{4} \Rightarrow \frac{1}{2}\cos x = \frac{\sqrt{2}}{4}$

$$\cos x = \frac{2\sqrt{2}}{4}$$

$$\cos x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

S | A ✓

T | C ✓

$$x = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

9E

12 (a) $h(x) = f(g(x))$ $\quad -\pi < x < \pi$

$h(x) = f(\cos x)$

$h(x) = 3\cos x - 2$

$h'(x) = -3\sin x$

(b) $2h'(x) = 3$

$\Rightarrow \quad -6\sin x = 3$

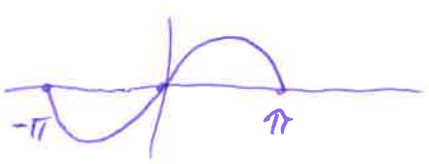
$\sin x = \frac{3}{-6}$

$\sin x = -\frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$

$x = 0 - \frac{\pi}{6}, \; -\pi + \frac{\pi}{6}$

$x = -\frac{\pi}{6}, \; -\frac{5\pi}{6}$



9E

(13) $f(x) = x + 8\cos x$

$f'(x) = 1 + -8\sin x$

$p(x) = 3 + f'(x)$

$= 3 + 1 - 8\sin x$

$p(x) = 4 - 8\sin x$

$q(x) = \frac{1}{x}$

$q(p(x))$

$= q(4 - 8\sin x)$

$= \frac{1}{4 - 8\sin x}$

$4 - 8\sin x \neq 0$

$4 \neq 8\sin x$

$\frac{4}{8} \neq \sin x$

$\sin x \neq \frac{1}{2}$

S ✓	A ✓
T	C

$x \neq \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x \neq \frac{\pi}{6}, \frac{5\pi}{6}$

$x \neq \frac{5\pi}{6}$ on $\frac{\pi}{2} < x < \pi$

9G

$$\frac{d}{dx}(f(x))^n = n f(x)^{n-1} \times \frac{d}{dx} f(x)$$

1(h) $\frac{d}{dx}(x-2)^6$

$= 6(x-2)^5 \times \frac{d}{dx}(x-2)$

$= 6(x-2)^5 \times 1$

$= \underline{6(x-2)^5}$

1(f) $\frac{d}{dx}(8(x-3)^6)$

$= 6 \times 8(x-3)^5 \times \frac{d}{dx}(x-3)$

$= 48(x-3)^5 \times 1$

$= \underline{48(x-3)^5}$

1(j) $\frac{d}{dx}(x-5)^{-4}$

$= -4(x-5)^{-5} \times \frac{d}{dx}(x-5)$

$= -4(x-5)^{-5} \times 1$

$= -4(x-5)^{-5}$

9G

1(v) $\frac{d}{dx}\left(\frac{4}{7x^3} + 8(x-4)^{-2}\right)$

$= \frac{d}{dx}\left(\frac{4x^{-3}}{7} + 8(x-4)^{-2}\right)$

$= \frac{-3 \times 4x^{-4}}{7} - 16(x-4)^{-3} \times \frac{d}{dx}(x-4)$ (1)

$= \frac{-12x^{-4}}{7} - 16(x-4)^{-3}$

$= \frac{-12}{7x^4} - \frac{16}{(x-4)^3}$

9G

2(f) $\frac{d}{dx}\left(10(6x+2)^3\right)$

$= 3 \times 10(6x+2)^2 \times \frac{d}{dx}(6x+2)$

$= 30(6x+2)^2 \times 6$

$= 180(6x+2)^2$

2(l) $\frac{d}{dx}(5x+4)^{-6}$

$= -6(5x+4)^{-7} \times \frac{d}{dx}(5x+4)$

$= -6(5x+4)^{-7} \times 5$

$= -30(5x+4)^{-7}$

$= \frac{-30}{(5x+4)^7}$

2(q) $\frac{d}{dx}\left[(x-4)^{-6} + \frac{4x^2-1}{x}\right]$

$= \frac{d}{dx}\left[(x-4)^{-6} + \frac{4x^2}{x} - \frac{1}{x}\right]$

9G

2(g) continued

$$\frac{d}{dx}\left[(x-4)^{-6} + 4x - x^{-1}\right]$$

$$= -6(x-4)^{-7} \times \frac{d}{dx}(x-4) + 4 + x^{-2}$$

$$= -6(x-4)^{-7} \times 1 + 4 + \frac{1}{x^2}$$

$$= -\frac{6}{(x-4)^7} + 4 + \frac{1}{x^2}$$

9G

3(g) $\frac{d}{dx}\frac{(2-3x)^4}{5}$

$= \frac{4(2-3x)^3}{5} \times \frac{d}{dx}(2-3x)$

$= \frac{4(2-3x)^3}{5} \times -3$

$= -\frac{12(2-3x)^3}{5}$

3(j) $\frac{d}{dx}\left(6x\sqrt{x} - \frac{(1-2x)^{-4}}{8}\right)$

$= \frac{d}{dx}\left(6x^1x^{1/2} - \frac{(1-2x)^{-4}}{8}\right)$

$= \frac{d}{dx}\left(6x^{3/2} - \frac{(1-2x)^{-4}}{8}\right)$

$= \frac{3}{2} \cdot 6x^{1/2} + 4\frac{(1-2x)^{-5}}{8} \times \frac{d}{dx}(1-2x)$

$= 9x^{1/2} + \frac{4(1-2x)^{-5}}{8} \times -2$

$= 9x^{1/2} - \frac{8^1(1-2x)^{-5}}{8_1} = 9\sqrt{x} - \frac{1}{(1-2x)^5}$

96

4(c) $\frac{d}{dx}\ \frac{1}{(4x+1)^5}$

$= \frac{d}{dx}\ (4x+1)^{-5}$

$= -5(4x+1)^{-6} \times \frac{d}{dx}\ (4x+1)$

$= -5(4x+1)^{-6} \times 4$

$= \frac{-20}{(4x+1)^6}$

4(h) $\frac{d}{dx}\ \frac{8}{(5x-1)^6}$

$= \frac{d}{dx}\ 8(5x-1)^{-6}$

$= -48(5x-1)^{-7} \times \frac{d}{dx}\ (5x-1)$

$= -48(5x-1)^{-7} \times 5$

$= -48 - 240(5x-1)^{-7}$

$= \frac{-240}{(5x-1)^7}$

Q4(l) $\frac{d}{dx}\left(\frac{2}{9(3x-2)^5}\right)$

$= \frac{d}{dx}\ \frac{2(3x-2)^{-5}}{9}$

$= \frac{-10(3x-2)^{-6}}{9} \times \frac{d}{dx}(3x-2)$

$= \frac{-10(3x-2)^{-6}}{9} \times 3$

$= \frac{-\overset{10}{\cancel{30}}(3x-2)^{-6}}{\cancel{9^3}}$

$= \frac{-10}{3(3x-2)^6}$

Q4(m) $\frac{d}{dx}\ \frac{1}{2-x}$

$= \frac{d}{dx}\ (2-x)^{-1}$

$= -(2-x)^{-2} \times \frac{d}{dx}(2-x)$

$= -(2-x)^{-2} \times -1$

$= (2-x)^{-2}$

$= \frac{1}{(2-x)^2}$

9G

5(k) $\frac{d}{dx}\sqrt[3]{(x-4)^2}$

$= \frac{d}{dx}(x-4)^{2/3}$

$= \frac{2}{3}(x-4)^{-1/3} \times \frac{d}{dx}(x-4)$

$= \frac{2}{3}(x-4)^{-1/3} \times 1$

$= \frac{2}{3\sqrt[3]{x-4}}$

5(n) $\frac{d}{dx}\frac{8}{\sqrt[4]{(x-3)^3}}$

$= 8(x-3)^{-3/4}$

$= -\frac{3}{4} \times 8(x-3)^{-7/4} \times \frac{d}{dx}(x-3)$

$= -6(x-3)^{-7/4} \times 1$

$= \frac{-6}{\sqrt[4]{(x-3)^7}}$

96

5(g) $\frac{d}{dx}\left(\frac{4}{\sqrt[5]{(5-2x)^3}}\right)$

$\frac{d}{dx}\left(\frac{4}{(5-2x)^{3/5}}\right)$

$= \frac{d}{dx} 4(5-2x)^{-3/5}$

$= -\frac{3}{5} \times 4(5-2x)^{-8/5} \times \frac{d}{dx}(5-2x)$

$= -\frac{12}{5}(5-2x)^{-8/5} \times -2$

$= \frac{24}{5}(5-2x)^{-8/5}$

$= \frac{24}{5(5-2x)^{8/5}}$

$= \frac{24}{5\sqrt[5]{(5-2x)^8}}$

9G

6(e) $\frac{d}{dx}(2x^2+5x-3)^{-1}$

$= -(2x^2+5x-3)^{-2} \times \frac{d}{dx}(2x^2+5x-3)$

$= -(2x^2+5x-3)^{-2} \times (4x+5)$

$= \frac{-(4x+5)}{(2x^2+5x-3)^2}$

6 (g) $\frac{d}{dx}\left(\frac{1}{x^2-5}\right)$

$= \frac{d}{dx}(x^2-5)^{-1}$

$= -(x^2-5)^{-2} \times \frac{d}{dx}(x^2-5)$

$= -(x^2-5)^{-2} \times 2x$

$= \frac{-2x}{(x^2-5)^2}$

9G

6(i) $\frac{d}{dx}\left(\frac{1}{\sqrt{2-3x^3}}\right)$

$= \frac{d}{dx}\left(2-3x^3\right)^{-\frac{1}{2}}$

$= -\frac{1}{2}\left(2-3x^3\right)^{-\frac{3}{2}} \times \frac{d}{dx}\left(2-3x^3\right)$

$= -\frac{1}{2}\left(2-3x^3\right)^{-\frac{3}{2}} \times -9x^2$

$= \frac{9x^2}{2}\left(2-3x^3\right)^{-\frac{3}{2}}$

$= \frac{9x^2}{2\sqrt{\left(2-3x^3\right)^3}}$

6(l) $\frac{d}{dx}\left(\sqrt{x}-2\right)^6$

$= 6\left(\sqrt{x}-2\right)^5 \times \frac{d}{dx}\left(\sqrt{x}-2\right)$

$= 6\left(\sqrt{x}-2\right)^5 \times \frac{d}{dx}\left(x^{\frac{1}{2}}-2\right)$

$= 6\left(\sqrt{x}-2\right)^5 \times \frac{1}{2}x^{-\frac{1}{2}}$

$= 3\left(\sqrt{x}-2\right)^5 x^{-\frac{1}{2}}$

$= \frac{3\left(\sqrt{x}-2\right)^5}{\sqrt{x}}$

9G

(8) $f(x) = \frac{1}{x-3}$

$f(x) = (x-3)^{-1}$

$f'(x) = -(x-3)^{-2} \times \frac{d}{dx}(x-3)$ (1)

$= \frac{-1}{(x-3)^2}$

$f'(0) = \frac{-1}{(0-3)^2}$

$= \frac{-1}{9}$

(11) $y = (x^2-2x-8)^4$

$\frac{dy}{dx} = 4(x^2-2x-8)^3 \times \frac{d}{dx}(x^2-2x-8)$

$= 4(x^2-2x-8)^3(2x-2)$

$\frac{dy}{dx} = 0 \Rightarrow 4(x^2-2x-8)^3(2x-2) = 0$

$x^2-2x-8 = 0$ $\quad$ $2x-2 = 0$

$(x-4)(x+2) = 0$ $\quad$ $2x = 2$

$x-4 = 0$ $\quad$ $x+2 = 0$ $\quad$ $x = 1$

$x = 4$ $\quad$ $x = -2$

9H

1(b) $\frac{d}{dx} \cos(5x)$

$= -\sin(5x) \times \frac{d}{dx}(5x)$ (5)

$= -5 \sin 5x$

1(e) $\frac{d}{dx} \cos\left(2x + \frac{\pi}{6}\right)$

$= -\sin\left(2x + \frac{\pi}{6}\right) \times \frac{d}{dx}\left(2x + \frac{\pi}{6}\right)$ (2)

$= -2 \sin\left(2x + \frac{\pi}{6}\right)$

1(i) $\frac{d}{dx}\left(\frac{1}{3} \sin(6x+2)\right)$

$= \frac{1}{3} \cos(6x+2) \times \frac{d}{dx}(6x+2)$ (6)

$= \frac{6}{3} \cos(6x+2)$

$= 2 \cos(6x+2)$

9H

1(m) $\frac{d}{dx}\left(\cos 5x - 2\sin 3x\right)$

$= -\sin 5x \times \frac{d}{dx}(5x) - 2\cos 3x \times \frac{d}{dx}(3x)$

$= -5\sin 5x - 6\cos 3x$

2(c) $y = \cos 3x$

$\frac{dy}{dx} = -\sin(3x) \times \frac{d}{dx}(3x)$

$= -3\sin(3x)$

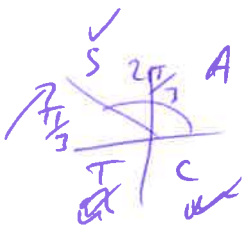
$\frac{dy}{dx}\left(\frac{2\pi}{9}\right) = -3\sin\left(3 \times \frac{2\pi}{9}\right)$

$= -3\sin\frac{2\pi}{3}$

$= -3\sin\pi/3$

$= -3 \cdot \frac{\sqrt{3}}{2}$

$= \frac{-3\sqrt{3}}{2}$



9H

2(e) $y = 6\cos\left(2x + \frac{\pi}{6}\right)$

$$\frac{dy}{dx} = -6\sin\left(2x + \frac{\pi}{6}\right) \times \frac{d}{dx}\left(2x + \frac{\pi}{6}\right)$$

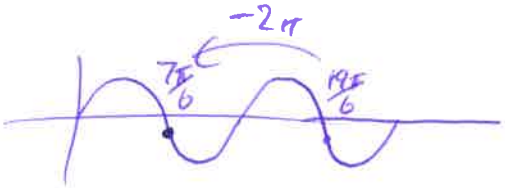
$$= -6\sin\left(2x + \frac{\pi}{6}\right) \times 2$$

$$= -12\sin\left(2x + \frac{\pi}{6}\right)$$

$$\frac{dy}{dx}\left(\frac{3\pi}{2}\right) = -12\sin\left(2\left(\frac{3\pi}{2}\right) + \frac{\pi}{6}\right)$$

$$= -12\sin\left(3\pi + \frac{\pi}{6}\right)$$

$$= -12\sin\left(\frac{19\pi}{6}\right)$$



$$= -12\sin\left(\frac{7\pi}{6}\right)$$

$$= 12\sin\left(\frac{\pi}{6}\right)$$

$$= 12 \times \frac{1}{2}$$

$$= \underline{6}$$

9H

④ (c) $y = 2\sin^3 x$

$= 2(\sin x)^3$

$= 6(\sin x)^2 \times \frac{d}{dx}\sin x$

$= \underline{6\sin^2 x \cos x}$

④ (f) $y = \frac{6}{5}\cos^5 x = \frac{6}{5}(\cos x)^5$

$\frac{dy}{dx} = 5 \times \frac{6}{5}(\cos x)^4 \times \frac{d}{dx}(\cos x)$

$= 6\cos^4 x \times -\sin x$

$= \underline{-6\cos^4 x \sin x}$

4(g) $y = (\sin x + \cos x)^3$

$\frac{dy}{dx} = 3(\sin x + \cos x)^2 \times \frac{d}{dx}(\sin x + \cos x)$

$= \underline{3(\sin x + \cos x)^2(\cos x - \sin x)}$

9H

4(h) $\frac{d}{dx}\left(\sin(2x-1)\right)^5$

$= 5\left(\sin(2x-1)\right)^4 \times \frac{d}{dx}\sin(2x-1)$

$= 5\left(\sin(2x-1)\right)^4 \times \cos(2x-1) \times \frac{d}{dx}(2x-1)$

$= 10\sin^4(2x-1)\cos(2x-1)$

(6) (a) $\frac{d}{dx}\frac{1}{\sin x}$

$= \frac{d}{dx}(\sin x)^{-1}$

$= -(\sin x)^{-2} \times \frac{d}{dx}\sin x$

$= -(\sin x)^{-2}\cos x$

$= \frac{-\cos x}{\sin^2 x}$

6H

6(c) $\frac{d}{dx}\sqrt{\sin x}$

$= \frac{d}{dx}(\sin x)^{1/2}$

$= \frac{1}{2}(\sin x)^{-1/2} \times \frac{d}{dx}\sin x$

$= \frac{1}{2}(\sin x)^{-1/2}\cos x$

$= \frac{\cos x}{2\sqrt{\sin x}}$

6(e) $\frac{d}{dx}\frac{1}{\cos(3x-\pi)}$

$= \frac{d}{dx}(\cos(3x-\pi))^{-1}$

$= -(\cos(3x-\pi))^{-2} \times \frac{d}{dx}\cos(3x-\pi)$

$= -(\cos(3x-\pi))^{-2} \times -3\sin(3x-\pi)$

$= \frac{3\sin(3x-\pi)}{\cos^2(3x-\pi)}$

6H

6(h) $\frac{d}{dx} \cos(\sin x)$

$= -\sin(\sin x) \times \frac{d}{dx}(\sin x)$

$= -\sin(\sin x) \times \cos x$

$= -\cos x \sin(\sin x)$

(9) $y = 2\sin 3x$

$\frac{dy}{dx} = 6\cos 3x$

$M = \frac{dy}{dx} = -3$ $\quad 6\cos 3x = -3$

$\cos 3x = -3/6$

$\cos 3x = -\frac{1}{2}$ $\quad 0 \le x \le \pi$

$\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

S	A
T	C

$3x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, \ldots$

$3x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots$ ($+2\pi$, $+2\pi$)

$x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}$ out of range

9H

(11) $3x + 2y = 0$

$2y = -3x$

$y = \frac{-3}{2}x \qquad m = -\frac{3}{2}$

$\frac{d}{dx}\cos(3x - \pi)$

$= -\sin(3x-\pi) \times \frac{d}{dx}(3x-\pi)$

$= -3\sin(3x-\pi)$

As parallel $\quad -3\sin(3x-\pi) = -\frac{3}{2}$

$\sin(3x-\pi) = \frac{-3}{2x-3}$

$\sin(3x-\pi) = \frac{1}{2} \qquad 0 \le x \le \pi$

S | A
T | C

$3x-\pi = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \ldots$

$3x-\pi = \frac{\pi}{6}, \overset{+2\pi}{\frac{5\pi}{6}}, \underset{+2\pi}{\frac{13\pi}{6}}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$3x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$

$x = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}$

check not outside of range $0 \le x \le 2\pi$ or need to look for value at other end of range (e.g. -2π)

$x = \frac{7\pi}{18}, \frac{11\pi}{18} \qquad 0 \le x \le \pi$